Separation Science and Technology
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713708471

Model for Predicting Filtration Efficiency and Pressure Drop in Axial Magnetic Filters

Saadetdin Herdem[a]; Muhammet Köksal[a]; Teymuraz Abbasov[a]
[a] DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING, ENGINEERING FACULTY, INONU UNIVERSITY, MALATYA, TURKEY

Online publication date: 16 May 2000

To cite this Article Herdem, Saadetdin , Köksal, Muhammet and Abbasov, Teymuraz(2000) 'Model for Predicting Filtration Efficiency and Pressure Drop in Axial Magnetic Filters', Separation Science and Technology, 35: 7, 941 — 950
To link to this Article: DOI: 10.1081/SS-100100203
URL: http://dx.doi.org/10.1081/SS-100100203

SEPARATION SCIENCE AND TECHNOLOGY, **35**(7), pp. 941–950, 2000

Model for Predicting Filtration Efficiency and Pressure Drop in Axial Magnetic Filters

SAADETDIN HERDEM, MUHAMMET KÖKSAL,* and TEYMURAZ ABBASOV

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING
ENGINEERING FACULTY
INONU UNIVERSITY
44069 MALATYA, TURKEY

ABSTRACT

The theory of filtration efficiency and filter impedance in axially ordered magnetic filters in conditions of laminar flow is described. The flow profile of the suspension that flows axially along the longitudinally ordered wires is determined by the Kuwabara–Happel cell model. Expressions for both filter impedance and filtration efficiency are obtained. In general, they are different from those predicted by previous filtration theories in ideal flow conditions. The derived theoretical formulas are simplified so they can be used easily in engineering applications. The results are compared with the experimental ones presented in the literature, and it is seen that they are consistent with each other.

Key Words. Magnetic filter; Magnetic separator; Filter impedance; Cell model

INTRODUCTION

High gradient magnetic filtration (HGMF) and separation is a method for the removal of micron-sized magnetic particles from suspensions. Fundamental to the magnetic separation process is the mechanism of particle capture (1–5). The theoretical and experimental investigation of the capturing mechanism of particles is essentially based on the model for particle capture and ac-

* To whom correspondence should be addressed.

cumulation on a single magnetized wire (6, 7). In this model, the so-called inviscid approximation, the fluid is considered to be ideal when it is interacting with the wires of the filter but assumed to be viscous when it is interacting with each of the particles. Magnetic filter and separator matrices are composed of ordered or arbitrarily arranged magnetized wires. An HGMF with a strictly ordered wire matrix of parallel ferromagnetic wires possesses a better efficiency than filters with an unordered matrix of, e.g., steel wool. Thus, there is a question of which matrix geometry and configuration gives optimal filter efficiency. The predictions of models that use the single wire approximation to describe efficiencies are often in disagreement with experimental data for two main reasons (8):

1. The single wire approximation can not account for effects determined by the wire arrangement in the matrix.
2. Filter efficiencies should ideally be measured for monosized spherical particles for which they have been calculated, but suspensions of such particles with suitable magnetic properties are difficult to obtain.

To obtain a more consistent theoretical expression for filter efficiency, it is necessary to consider the flow velocity profile of the suspension through the matrix elements and the packing fraction of the wires. The effect of the location of the wires for different configurations in the matrix on the efficiency of magnetic filters and separators has been investigated by some authors (2–5, 8–10). Since the results obtained so far are mathematically complex, they are not convenient for engineering purposes. The derivation of more satisfactory expressions to show the dependency of filtration efficiency and pressure drop on filtration system parameters is important for design and other engineering purposes.

In this work the deposition of fine magnetic particles carried by suspension on magnetized ferromagnetic wires with a certain volume packing fraction is investigated. The flow velocity profile of the suspension flowing axially through the wires is determined by the Kuwabara–Happel cell model (11). The relationships needed to determine the cleaning efficiency and the impedance for filters consisting of magnetized axially located wires are obtained. The obtained results are compared with similar ones derived by other methods and with experimental results given in the literature.

LAMINAR FLOW MODEL OF PARTICLE CAPTURE AND FILTER EFFICIENCY OF AXIAL MAGNETIC FILTERS

Consider a linear ferromagnetic cylindrical wire of radius a magnetized orthogonally to its symmetry axis by an external magnetic field of intensity H_0 (Fig. 1). A suspension of radius δ carrying the magnetic particles is assumed flowing parallel to the wire with a velocity V_0.

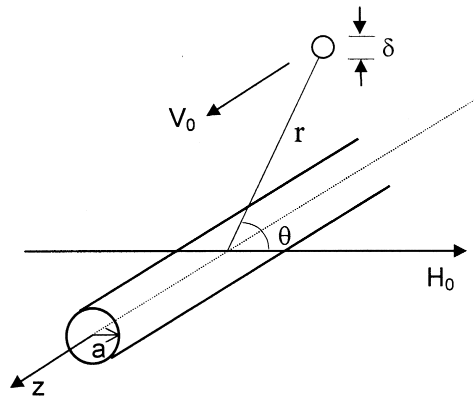

FIG. 1 Axial configuration of the linear wire, magnetic particle, and magnetic field.

From a hydrodynamical point of view, the flow profile of the liquid along the wire is assumed to vary in a single cell (11). Kuwabara and Happel used this model in an attempt to approximate the parallel and transverse flows in a double periodic lattice of cylinders. Both defined a cell by assuming the existence of an idealized concentric cylinder of radius b about each cylinder of radius a, where the radius b is chosen to satisfy

$$\gamma b^2 = a^2 \tag{1}$$

so that the volume fraction of the cylinders in the system (γ) is maintained. According to this model, the axial velocity profile of the liquid flow over the cylinder is determined by (11)

$$V_z(r) = \frac{V_0(1-\gamma)\gamma}{2\text{Ku}}\left[1 - \left(\frac{r}{a}\right)^2 + \frac{2}{\gamma}\ln\frac{r}{a}\right] \tag{2}$$

Here r is the radial coordinate. Ku is known as Kuwabara's constant and is equal to

$$\text{Ku} = \gamma - 0.5\ln\gamma - 0.25\gamma^2 - 0.75 \tag{3}$$

In the case of a laminar flow rate of Newtonian liquid carrying the particles, the efficiency of the axial filter can be obtained by a modification of the method proposed by Birss et al. (2, 3) and Gerber (4). Let N be the number of magnetic particles per unit volume of the fluid at the inlet of the filter. Consider a unit area of the filter cross section: the number of particles entering per unit time is $N_{in} = N\langle V\rangle$, where $\langle V\rangle$ is the average velocity of the fluid in the filter. The number of particles coming out of the filter per unit cross-sectional area per unit time is $N_{out} = N_{ne}\langle V_e\rangle$, where N_{ne} is the number of the noncaptured particles per unit volume of the fluid in the filter and $\langle V_e\rangle$ is the average escape velocity. The probability that a particle will not be captured is

$$(1 - a_f \pi a^2 r_{ca}^2)^n \tag{4}$$

where $r_{ca} = r_c/a$ is the normalized capture radius, and consequently

$$N_{ne} = (1 - a_f \pi a^2 S_a^*)^n N \tag{5}$$

Here a_f is a measure of the noncircularity of the capture cross section, n is the number of wires per unit area of the filter cross section, and S_a^* is the normalized capture cross section (6). In accordance with the condition $a_f \gamma S_a^* \ll 1$, the formula for filter efficiency can be written as

$$\frac{N_{out}}{N_{in}} = \frac{\langle V_e \rangle}{\langle V \rangle} \exp(-\gamma a_f S_a^*) \tag{6}$$

As all quantities in the argument of the exponential function are known, it remains to determine the velocity correction ratio $\langle V_e \rangle / \langle V \rangle$.

To determine this velocity ratio, the flow of the fluid in both of the capture and escape regions must be investigated. It is assumed that the capture radius r_c is limited to the region where the wire affects the velocity profile, i.e., $a < r_c < b$. In the capture and escape regions the average velocity of the fluid is determined by

$$\langle V \rangle = \frac{2\pi \int_a^{r_c} vr\, dr}{\pi (r_c^2 - a^2)} \tag{7}$$

$$\langle V_e \rangle = \frac{2\pi \int_{r_c}^{b} vr\, dr}{\pi (b^2 - r_c^2)} \tag{8}$$

respectively. First, it can be assumed that the velocity is approximately steady around the boundary of the cell, i.e., $\langle V_e \rangle \approx V_0$. Considering Eq. (2), taking the integral in Eq. (7), and inserting the result in Eq. (6), the following ratio is obtained:

$$\frac{N_{out}}{N_{in}} = \frac{2\mathrm{Ku}}{(1-\gamma)(\gamma - 1 - \ln \gamma)} \exp(-a_f \gamma S_a^*) \tag{9}$$

As a result of many experimental and theoretical results about the magnetic filters and separators, the following approximation is obtained (3, 6):

$$S_a^* \approx \frac{4}{\pi} L_a^{0.5} \tag{10}$$

where $L_a = (V_m/V_0)(L/a)$ is the normalized filter capture length (12), L is the filter length, and V_m is the magnetic velocity (1). Using Eqs. (1) and (10), the

Marcel Dekker, Inc.
270 Madison Avenue, New York, New York 10016

above equation can be written in the form

$$\frac{N_{\mathrm{out}}}{N_{\mathrm{in}}} = \alpha(\gamma)\exp\left(-\frac{4\gamma a_{\mathrm{f}}}{\pi} L_{\mathrm{a}}^{0.5}\right) \tag{11a}$$

where

$$\alpha(\gamma) = \frac{2\mathrm{Ku}}{(1-\gamma)(\gamma - 1 - \ln\gamma)} \tag{11b}$$

These equations can be used to determine the filter efficiency by

$$\psi = 1 - \frac{N_{\mathrm{out}}}{N_{\mathrm{in}}} \tag{12}$$

The filter efficiency obviously depends on the geometric, magnetic, and hydromechanic parameters of the system. These parameters are expressed in terms of the normalized filter length L_{a}.

In the case of a Newtonian ideal flow regime, by assuming $V_{\mathrm{z}}(r) = V_0$ a similar procedure yields

$$\psi = 1 - \exp\left(-\frac{4\gamma a_{\mathrm{f}}}{\pi} L_{\mathrm{a}}^{0.5}\right) \tag{13}$$

for the filter efficiency.

Experimental results for the efficiency of a filter of length $L = 0.1$ m are shown against the packing fraction up to 14% (3) in Fig. 2. The liquid filtered

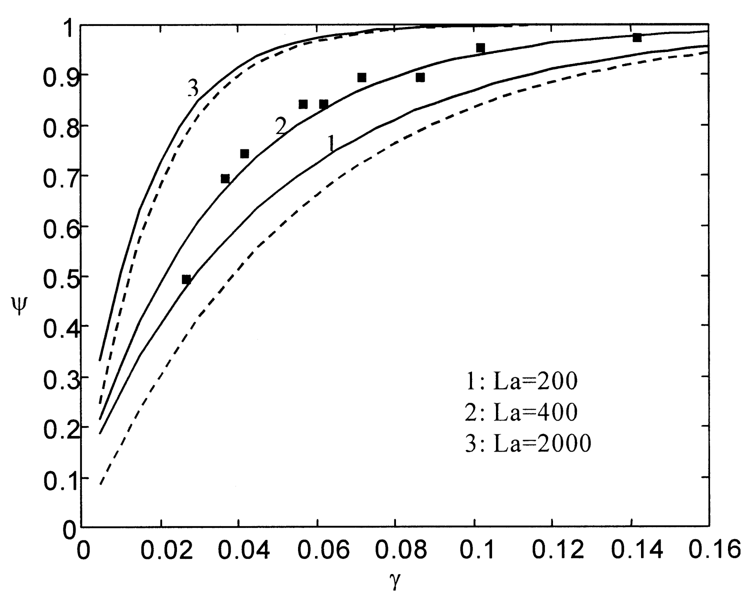

FIG. 2 Variation of filter efficiency as a function of packing fraction γ for ideal (- - -) and laminar (——) flows.

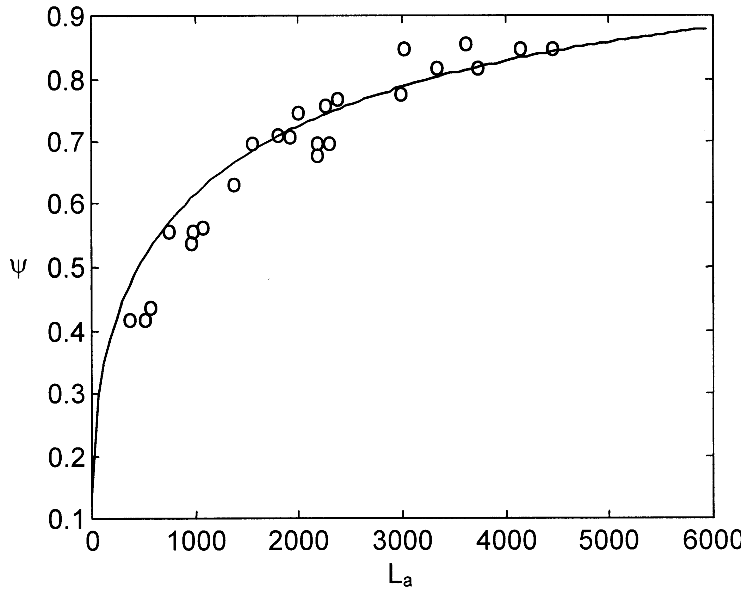

FIG. 3 Theoretical and experimental values of the filter efficiency for $\gamma = 1.6\%$ as a function of normalized filter length.

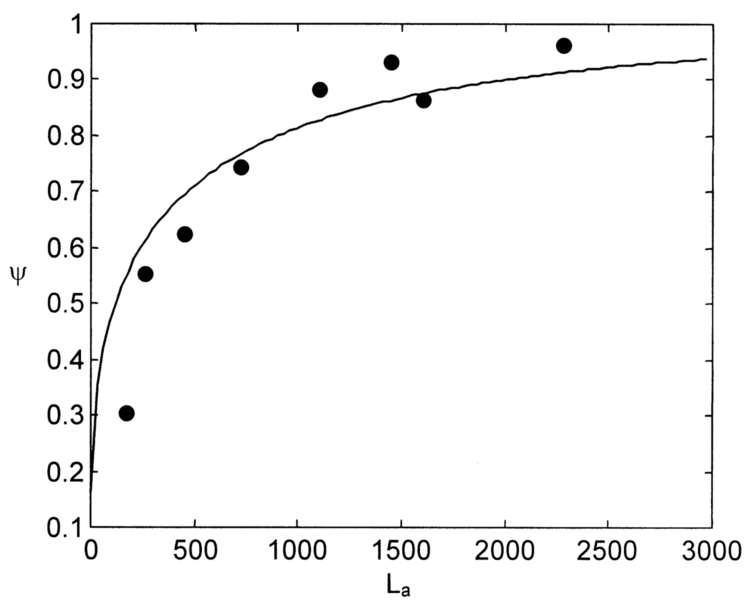

FIG. 4 Theoretical and experimental values of the filter efficiency for $\gamma = 3\%$ as a function of normalized filter length.

is a quasi-colloidal suspension with CuO particles, and the filtration velocity is $V_0 = 10^{-2}$ ms^{-1}. In the same figure the filter efficiency calculated theoretically from Eq. (11) with $L_a = 200$, 400, and 2000 is plotted. The experimental results are compared with the theoretical results computed from Eq. (11) for the filter used in the experiments [$L_a = 400$ (3)]. The experimental results are obviously in good agreement with the theoretical results.

In Figs. 3 and 4 the dependency of the filter efficiency on the normalized filter length is shown for different volume packing fractions of the matrix elements. The experimental results taken from the literature (8) are indicated in the same figures. The experimental results are in good agreement with the theoretical results obtained from Eqs. (11a) and (11b).

DETERMINATION OF SPECIFIC FLUID IMPEDANCE

Assuming that the fluid is incompressible and using the Kuwabara–Happel cell model, the total flux through the cell and the specific fluid impedance ρ of the unloaded filter can be determined. The specific fluid impedance of the unloaded filter is defined in Refs. 3 and 4 as

$$\rho = \left(\frac{\Delta P}{L}\right)\frac{S}{Q} \tag{14}$$

where S is the cross-section area of the multiwire filter and Q is the fluid discharge (in m^3/s). Note that all the quantities in the right-hand side of this equation are measurable.

The discharge Q can also be given as

$$Q = nSQ_w \tag{15a}$$

where

$$Q_w = a^3 \int_0^{2\pi} \int_1^{b_a} r_a V_z(r)\, dr_a\, d\phi \tag{15b}$$

is the discharge associated with one representative wire. Inserting the expression in Eq. (2) for $V_z(r)$ into the above equation, Q_w is obtained as

$$Q_w = \pi a^4 \left(\frac{\Delta P}{2\eta L}\right)\gamma \mathrm{Ku} = \langle V\rangle\,(1-\gamma)\pi b^2 \tag{16}$$

Computing Q from Eqs. (15a) and (16) and inserting the result in Eq. (14), the specific impedance is obtained as

$$\rho = \frac{2\eta}{a^2}\frac{\gamma}{\mathrm{Ku}} \tag{17}$$

Considering Eq. (3), this expression can be written as

$$\rho = \frac{8\eta}{a^2} \frac{1}{4 - \gamma - \frac{1}{\gamma}(2 \ln \gamma + 3)} \tag{18}$$

This equation is identical with the one which Birss et al. (2, 3) obtained for axial filters and for Newtonian fluid flow by using the Wigner–Seitz cell model that involves some mathematical approximations. They have also recorded that this formula is well justified by the experimental results. This fact indicates that the Kuwabara–Happel cell model makes it possible to obtain a more convenient mathematical expression for the specific impedance.

It is possible to obtain other expressions for the specific impedance of the filter which is very important for HGMF regimes from a practical point of view. For this aim, the following expressions of the pressure drop on the fibrous filters can be used (11): The general expression of the pressure drop obtained from the Kuwabara–Happel cell model is

$$\frac{\Delta P}{L} = \frac{2V_0\eta\gamma(1 - \gamma)(5 - \gamma)}{3a^2\mathrm{Ku}} \tag{19a}$$

and Davies' experimentally correlated formula is

$$\frac{\Delta P}{L} = \frac{16V_0\eta}{a^2} \gamma^{1.5}(1 + 56\gamma^3) \tag{19b}$$

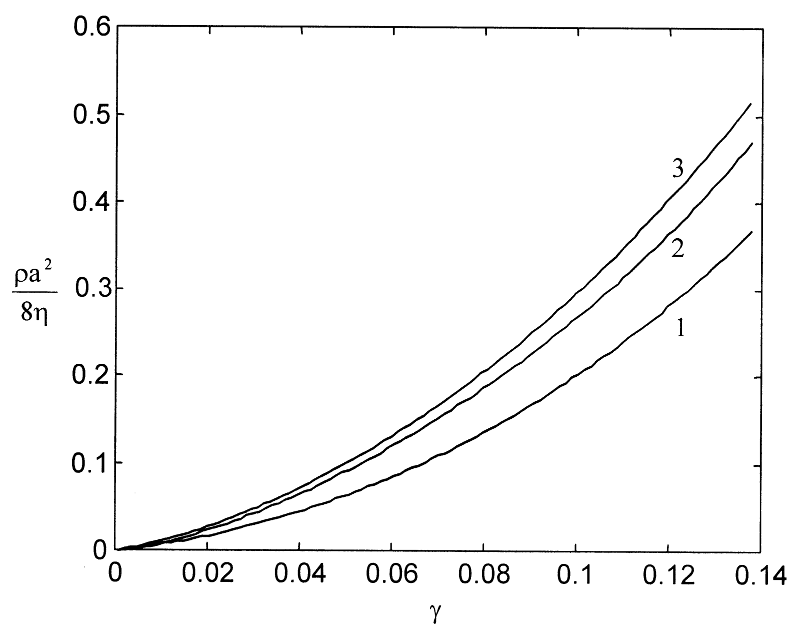

FIG. 5 Normalized filter specific impedance as a function of wire packing fraction γ. 1: From Eq. (18); 2: from Davies' experimentally correlated formula (Eq. 20b); and 3: from Eq. (20a).

Using these expressions and carrying out the simple mathematical operations in Refs. 3 and 4, the following expressions for the filter impedance can be obtained from Eqs. (19a) and (19b):

$$\rho = \frac{2\eta}{a^2}\frac{\gamma(1-\gamma)(5-\gamma)}{3\mathrm{Ku}} \tag{20a}$$

$$\rho = \frac{2\eta}{a^2}8\gamma^{1.5}(1+56\gamma^3) \tag{20b}$$

The dependency of the dimensionless filter impedance in Eqs. (18), (20a), and (20b) on the wire packing fraction is shown in Fig. 5. As observed in the figure, all three formulas give similar results. However, the specific filter impedance obtained in Eq. (20a) by the Kuwabara–Happel cell model and the one obtained in Eq. (20b) by the Davies formula are more consistent. These results prove the correctness of the expressions obtained by the Kuwabara–Happel cell model and their practical usage in engineering applications.

CONCLUSIONS

The technological and regime parameters of HGMF are determined by using the Kuwabara–Happel cell model. A simple expression of filter efficiency to clean liquids from the magnetic particles by HGMF is obtained. This model also makes it possible to determine a correct analytical expression for the specific filter impedance, which is another important parameter for the theory and application of HGMF. Moreover, investigation of HGMF theory from the laminar flow point of view permits consideration of the real properties of the liquid. Thus, the results can be directly compared with experiment. The laminar flow theory described here is in good qualitative agreement with experiment.

NOMENCLATURE

a	radius of matrix elements (cylinders, m)
a_f	correlation coefficient
b	cell radius in Kuwabara–Happel model (m)
Ku	Kuwabara's constant
L	filter length (m)
L_a	normalized filter length
n	number of wires per unit area of the filter cross section
ΔP	pressure drop on filter (Pa)
r	radial polar coordinate (m)
r_c	particle capture radius (m)
S_a^*	normalized capture cross section

V_0 flow velocity of suspension (m/s)
$\langle V \rangle$ average flow velocity in filter (m/s)
$\langle V_e \rangle$ average escape flow velocity (m/s)
V_m magnetic velocity (m/s) (1)

Greek Letters

$\alpha(\cdot)$ function defined by Eq. (11b)
γ wire volume packing fraction
δ particle size (diameter, m)
η dynamic viscosity (Ns/m^2)
ρ specific filter impedance ($Ns \cdot m^{-4}$)
ψ filter efficiency

REFERENCES

1. J. H. P. Watson, *J. Appl. Phys.*, *44*(9), 4209 (1973).
2. R. R. Birss, R. Gerber, M. R. Parker, and T. J. Sheerer, *IEEE Trans. Magn.*, *MAG-14*(5), 389 (1978).
3. R. R. Birss, R. Gerber, and M. R. Parker, *Ibid.*, *MAG-14*(6), 1165 (1978).
4. R. Gerber, *J. Phys. D.*, *11*(15), 2119 (1978).
5. V. Badescu, O. Rotariu, V. Murrariu, and N. Rezlescu, *Int. J. Multiphase Flow*, *22*(4), 797 (1996).
6. S. Uchiyama, S. Kondo, M. Takayasu, and I. Eguchi, *IEEE Trans. Magn.*, *MAG-12*, 895 (1976).
7. C. Cowen, F. C. Fredlaender, and R. Jaluria, *Ibid.*, *MAG-12*, 466 (1976).
8. H. Greiner, G. Reger, and H. Hoffmann, *Ibid.*, *MAG-20*(5), 1171 (1984).
9. R. Gerber, P. Krist, and L. Tarrant, *Ibid.*, *32*, 5100 (1996).
10. M. Franz and M. Franzreb, *Ibid.*, *34*(6), 3902 (1998).
11. D. J. Banks, *Part. Sci. Technol.*, *5*(3), 339 (1987).
12. R. Gerber and R. R. Birss, *High Gradient Magnetic Separation*, Wiley, Chichester, 1983.

Received by editor February 16, 1999
Revision received October 1999